Book Reviews

Nonlinear Dynamics and Chaos,, by J.M.T. Thompson and H.B. Stewart, John Wiley & Sons, New York, 1986, 376 pp., $42.95.

This book will be welcomed by all those readers who have sought an introduction to the recent developments in nonlinear dynamics and chaos. The recent discovery (or rediscovery some would say, giving Poincaré his due) of random-like, broad-band oscillations in the response of simple mechanical systems whose parameters are entirely deterministic, has led to one of the most exciting periods in the development of nonlinear dynamics since the days of Poincaré. There are some who contend that Poincaré understood chaos (and perhaps he did), but it is only with the advent of modern computational power that our knowledge of chaos has deepened and its presence discovered to be so pervasive in many dynamical systems of physical interest. Classical analytical methods which assume the system motion to be periodic obviously fail, and even more modern geometric approaches only hint at the possibility of chaos. For example, see the text by Guckenheimer and Holmes.[1]

Thus it is not surprising that the authors of the present book have relied upon modern geometrical concepts, substantial digital simulation of the dynamic response of nonlinear systems, and graphical presentation of data to describe the new results in nonlinear dynamics. The approach is quite different from Guckenheimer and Holmes who tried valiantly to prove or discuss as many relevant theorems and lemmas as might be useful. The approach is also distinct from the forthcoming book by F.C. Moon,[2] which the reviewer has had the opportunity to read in manuscript. Moon's book emphasizes the physical phenomena and basic conceptual framework, including an authoritative discussion of laboratory experiments. The three volumes of Guckenheimer and Holmes, Thompson and Stewart, and Moon thus span a spectrum of approaches to the subject and are complementary to one another. The reviewer would recommend all three to the reader with the suggestion that the present book or Moon's might be the best place to start.

One of the fundamental challenges any book on nonlinear dynamics and chaos faces is that our understanding of the subject is advancing so rapidly that any book risks being out-of-date shortly after its publication. This leads to some of the reservations the reviewer has about the present volume which will be outlined below. Even so, one may ask how well have the authors done with their book given this inevitable difficulty?

The book is generally clearly written, although one does detect a significant variation in style and interests between the two authors. Thus, readers may have a difference in opinion as to which part of the book they like best. The subjects covered are standard, or as nearly standard as one might expect in a rapidly evolving field. The organization of the book is generally logical and most readers will be able to readily find their starting point in the book based upon their own background. They also can easily decide which portions to omit, at least on a first reading, based upon their own interests. All in all, the authors have done a professional job. The graphics are particularly good, which is especially important for this subject.

Neverthless, one can offer constructive criticisms, both broadly and in detail. First, there are some important topics which are omitted altogether or only lightly touched upon:

What is missing in the book? A watershed event in the subject has been the first quantitative correlation between theory and experiment for the onset of chaos. This is discussed in a recent article by Dowell and Pezeshki,[3] based upon their own calculations and the earlier physical experiment by Moon[4] on a buckled elastic beam which is periodically forced. Although the present authors discuss the earlier theoretical results of Holmes and their own, there is no discussion of the more recent and comprehensive theoretical results and the comparison with Moon's experiment. Perhaps the publication date deadline for the book prevented this discussion. This omission is especially unfortunate in that the buckled beam problem offers a particularly clear physical example to introduce and discuss virtually all the basic concepts of nonlinear dynamics and chaos.

Fractal behavior and dimension. Although mentioned, this important set of concepts is only briefly discussed and the reader must look elsewhere for a more comprehensive treatment. See, for example, the book by Moon.[2]

Lyapunov exponents. The authors touch upon this and even present results of calculations (see their Fig. 15.10), however their discussion omits any clear description of how a Lyapunov exponent is defined or to be calculated, the authors being content to reference the literature, e.g., Wolf et al.

Necessary and sufficient conditions for chaos. Another significiant omission, and understandably so since the literature is still incomplete, is a definitive discussion of necessary and sufficient conditions for chaos. For a more extensive discussion of several such conditions, the reader may wish to consult the forthcoming survey article in *Computational Mechanics* by Dowell.[5] A recent contribution of Dowell and Pezeshki[6] has produced a definitive necessary and sufficient condition for the onset of chaos in Duffing's equation and related systems. Earlier suggestive work was done by Ueda.[7] Ueda's work is discussed by Thompson and Stewart in Chap. 13.

Second, some more detailed observations on the authors' presentation may be helpful to the reader. These are given below (in order of chapter):

1. *Introduction:* The choice of the Ueda version of the Duffing equation, with the linear stiffness term omit-

ted, is understandable from an historical point of view (Ueda is one of the most important contributors to the field), but unfortunate from a pedagogical point of view. The full Duffing equation (later treated in Chap. 6) would have been a better choice.

2. *An Overview of Nonlinear Phenomena:* A standard review. It may leave the impression with some readers that chaos will not occur in a Duffing equation with positive linear stiffness. This is subsequently rectified in Chap. 5, p. 71.

3. *Point Attractors in Autonomous Systems:* The richness and importance of the initial value problem deserves more attention than it receives here. Though ideas such as basin boundaries or domains of attraction are mentioned throughout the book, the discussion is curiously understated.

4. *Limit Cycles in Autonomous Systems:* A standard review which is generally well done.

5. *Periodic Attractors in Driven Oscillators:* The important notions of Poincaré maps and the variational equation for subharmonics are introduced.

6. *Chaotic Attractors in Forced Oscillators:* A descriptive treatment. This might have been Chap.1.

7. *Stability and Bifurcation of Equilibrium and Cycles:* A nice discussion of these topics.

8. *Stability and Bifurcation of Maps:* A discussion of linear stability theory and elements of castastrophe theory. What is missing from this book, and indeed not available anywhere in the literature, is a systematic way of generating maps from flows (differential equations). Until this gap is filled, maps are, to use a phrase quoted by the authors, "models of models" whose physical significance is problematical. This has not prevented some investigators from comparing the results of *map* theory with *flow* experiment. See Chap. 17 of this book.

9. *Chaotic Behavior of One- and Two-Dimensional Maps:* The title says it all.

10. *The Geometry of Recurrence:* A general discussion of the geometric structure of dynamical systems and their recurrence properties.

11. *The Lorenz System:* A brief review of this historically important system. The authors carefully skirt the crucial point made by Sparrow[8] in his definitive treatise on the Lorenz equations, that these equations represent a truncated and unconverged approximation to a set of partial differential equations (the Navier-Stokes equations). Therefore there is no hope of comparing the theoretical results from the Lorenz system with experiment for the physical system they were originally intended to represent.

12. *Rossler's Band:* A favorite, relatively simple, system for trying to do something analytically with chaos. It is not clear whether much of this can be extended to more interesting physical systems.

13. *Geometry of Bifurcations:* An effort to classify and identify routes to chaos. The authors give a necessarily incomplete account reflecting the known state of the art as of their writing. It is one of the outstanding issues and the authors have made a good effort. For an alternative approach, see Refs. 5 and 6.

14. *Subharmonic Resonances of an Offshore Structure:* An interesting physical application and one on which Professor Thompson has extensively published.

15. *Chaotic Motion of an Impacting System:* A limiting case of Chap. 14 and one on which others, notably Holmes and Shaw, have published as well. Because the system is bilinear, futher analytical progress can be made than is usual.

16. *The Particle Accelerator and Hamiltonian Dynamics:* Particle physicists often find it convenient and apparently useful to omit damping from their models and study the consequences of having a conservative system with many (sometimes only two!) degrees of freedom. This chapter considers such a model. For mechanical systems the omission of damping is generally a fundamentally fatal flaw.

17. *Experimental Observations of Order and Chaos:* This chapter is a contribution by Professor H.L. Swinney and is a modest revision of his earlier article in *Physica D* (1983). A discussion of a collection of four physical systems is given: an electrical circuit, a chemical reactor, Rayleigh-Bernard convection, and Couette-Taylor flow. Unfortunately, not one of these systems contain a theoretical model based upon first principle (e.g., a field theory from a variational principle or partial differential equation) from which reliable quantitative results have been obtained. This is unlike the buckled beam problem (a system of Duffing equations is a direct consequence of classical beam theory derived from the theory of elasticity) for which reliable theoretical results have been obtained and the results compared with those of a physical experiment.[3,4] This latter work was mostly done after 1983 and, perhaps, that is why it is not discussed here.

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[1]Guckenheimer, J. and Holmes, P., *Nonlinear Oscillations, Dynamical Systems, and Bifurcations to Vector Fields,* Springer-Verlag, Inc., New York, 1983.

[2]Moon, F.C., *Experiments in Chaotic Vibrations,* John Wiley and Sons, New York, 1987.

[3]Dowell, E.H. and Pezeshki, C., "On the Understanding of Chaos in Duffings Equation Including a Comparison with Experiment," *Journal of Applied Mechanics,* Vol. 53, 1986, pp. 5-9.

[4]Moon, F.C., "Experiments on Chaotic Motions of a Forced Nonlinear Oscillator: Strange Attractors," *Journal of Applied Mechanics,* Vol. 47, 1980, pp. 638-644.

[5]Dowell, E.H., "Chaotic Oscillations in Mechanical Systems," invited paper to appear in *Computational Mechanics,* 1987.

[6]Dowell, E.H. and Pezeshki, C., "On Necessary and Sufficient Conditions for Chaos to Occur in Duffings Equations,"*Journal of Sound and Vibration,* submitted for publication.

[7]Ueda, Y., "Explosion of Strang Attractors Exhibited by Duffings Equation," *Nonlinear Dynamics,* edited by Robert H.G. Helleman, New York Academy of Sciences, New York, 1980, pp. 422-434.

[8]Sparrow, C., *The Lorenz Equations: Bifurcations, Chaos and Strange Attractors,* Applied Mathematical Sciences, Vol. 41, Springer-Verlag, New York, 1982.